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Computer-Aided Eulerian Air Traffic Flow Modeling and Predictive Control

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Eulerian models are used to represent the air traffic environment as traffic flows between interconnected control volumes representing the airspace system. Although these models can be manually derived for simple air traffic patterns, computer-based approaches are essential for modeling realistic airspaces involving multiple traffic streams. Starting from the specification of a few airspace parameters and traffic data, the developed computer-aided modeling technique can automatically construct Eulerian models of the airspace. The synthesis of air traffic flow control algorithms using the model predictive control technique in conjunction with these models is given. It is shown that the flow control logic synthesis can be cast as a linear programming problem. The flow control methodology is illustrated using air traffic data over two regions in U.S. airspace.

I. Introduction

THE development of an Eulerian (see Ref. 1) approach to modeling air traffic was discussed in recent research efforts (see Refs. 2 and 3). These works were motivated by research initiatives currently underway[4,5] within the air traffic management (ATM) research community to develop decision support tools for analyzing and controlling air traffic flow, to manage more efficiently operations of the U.S. National Airspace System (NAS). The focus of the present paper is on the development of a computer-aided methodology for deriving Eulerian models of the airspace and employing it for air traffic flow control. The approach uses the NASA-developed Future ATM Concepts Evaluation Tool[6] (FACET) software as its foundation.

The Eulerian approach models the airspace in terms of line elements approximating airways, together with merge and diverge nodes. Because this modeling technique spatially aggregates the air traffic, the order of the airspace model depends only on the number of line elements used to represent the airways and not on the number of aircraft operating in the airspace. Eulerian models are in the form of linear, time-varying difference equations.

A one-dimensional modeling methodology is an intuitive approach for deriving models of traffic flow networks formed by jet routes and Victor airways. However, not all aircraft in the airspace strictly follow the jet routes or Victor airways. This situation is likely to continue in the future as more aircraft opt to fly wind-optimal routes to their destinations.[7] This introduces the need for a more flexible modeling framework. This framework, first advanced in Ref. 2, discretizes the airspace into surface elements (SEL), within which the traffic flow is aggregated into eight different directions. This modeling provides adequate fidelity in en route airspace in which the traffic flow is largely two dimensional. The traffic at all flight levels in class A airspace [at or above 18,000 ft (5500 m)] is classified as belonging to any one of these eight directions, with inflows and outflows from airports and other external sources. Each SEL is connected to its eight neighbors, with the connection strengths being determined by the actual traffic flow patterns.

Eulerian models are then derived by examining traffic flows into and in between the SELs over a specified sample time interval. These models are then used for analysis and flow control system design. Details of the modeling approach will be given in Sec. II. It has been shown in Refs. 2 and 3 that the Eulerian models can be used to carry out a variety of analyses on the air traffic flow, such as controllability, reachability, and model decentralization.

An important application of the Eulerian models is in development of quantitative decision support tools for air traffic flow control. Very little research has been conducted on the use of automatic control theory for the ATM problem.[7,8] The present research represents an initial attempt and explores the application of the model-predictive control (MPC) technique[9–11] to the air traffic flow control problem. Alternate control approaches are applicable to this problem. This will be explored in future research efforts. Section III presents a discussion of the air traffic flow control using the MPC technique, together with two examples. Conclusions from the present research are given in Sec. IV.

II. Computer-Aided Eulerian Air Traffic Flow Modeling

The Eulerian modeling process begins with the definition of a grid of SELs covering the region of airspace being modeled. The SEL grid is defined by latitude–longitude tessellation on the surface of the Earth in geocentric polar coordinates. Each surface element has equal angular dimensions in longitude and latitude as shown in Fig. 1. However, due to the spherical nature of the airspace being modeled, SELs far north or south of the equator will have smaller physical dimensions than those near the equator. All of the results reported in this paper are based on 1° latitude–longitude increments. The eight different en route traffic flow directions within each SEL are indicated in Fig. 2. In addition to these, the SELs above airports will include one output stream for landing aircraft. The aircraft taking off from airports under an SEL are included in one of the eight en route traffic flow directions. SELs lying on the boundary of the airspace being modeled will have additional inputs representing

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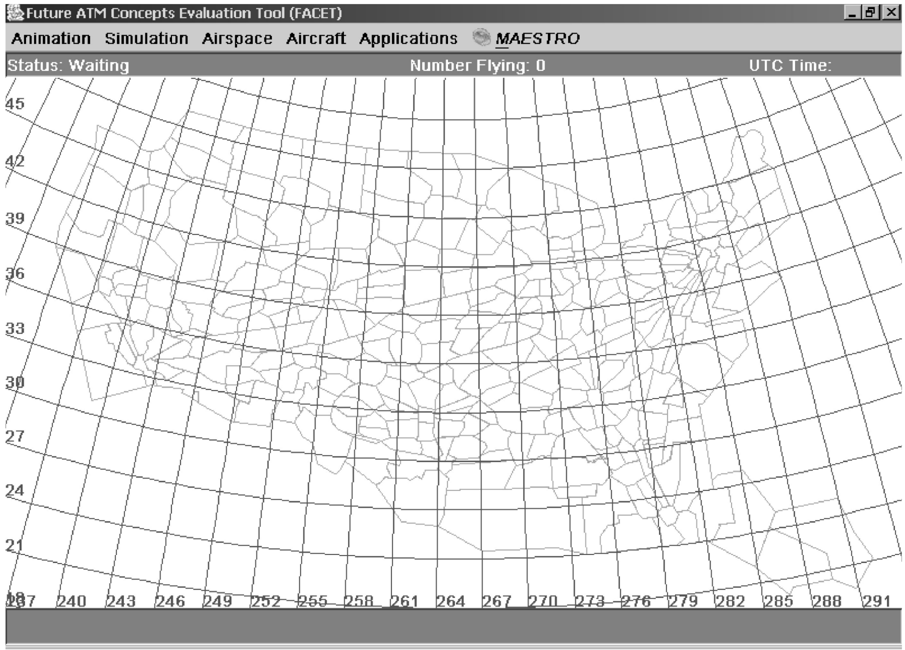

Fig. 1 Latitude–longitude tessellation used in Eulerian flow modeling.

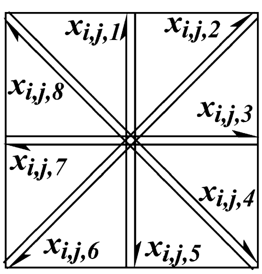

Fig. 2 Traffic flow directions in an SEL (*i*, *j*).

traffic entering the system from unmodeled airspace, for example, international flights.

Because the present form of the Eulerian model is discrete in space and time, a sample interval τ must also be specified. Although the spatial and temporal discretizations are based mainly on the level of detail desired in the model, due to the assumption that each SEL is connected to only eight of its neighbors, the sample time interval must be chosen so that no aircraft in an SEL travels beyond its immediate neighbors in a sample interval. Thus, the dimensions of the smallest SEL and the airspeed of the fastest aircraft in the airspace determine the acceptable sample interval.

As in Refs. 2 and 3, the air traffic flow pattern is modeled within each SEL using two sets of parameters. The first of these are the inertia parameters $a_{ijmm}(k)$, where the first two subscripts denote the particular surface element $(i,\ j)$, the second two subscripts denote the stream numbers within that element, and the dependence on k denotes the time interval. There is one parameter for each of the eight streams representing the fraction of the aircraft that remained in the SEL from the previous sample time. By definition, in any stream i, the fraction of aircraft that left the SEL in the previous sample interval is given by $[1 - a_{ijmm}(k)]$.

The second set of parameters contains the flow divergence parameters $\beta_{ijmn}(k)$ representing the aircraft that switched streams from m to n within the SEL $(i,\ j)$ in the time interval k. Because the aircraft in a stream may stay in it, or switch to any of the other seven en route streams, or land at an airport, for a given SEL there is a matrix of $9 \times 8 = 72$ flow divergence parameters. To satisfy the principle of conservation of aircraft in an SEL, for each stream n the divergence parameters to all of the outputs must add up to unity, that is,

$$\sum_{m=1}^{9} \beta_{ijmn}(k) = 1 \tag{1}$$

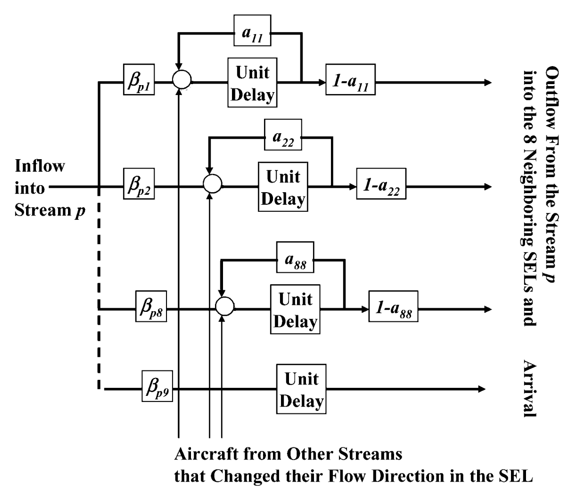

Fig. 3 Eulerian model of an air traffic stream in an SEL.

Note that for each n, one of the $\beta_{ijmn}(k)$ is not independent. By convention, let

$$\beta_{ijnn}(k) = 1 - \sum_{\substack{m=1 \\ m \neq n}}^{9} \beta_{ijmn}(k) \tag{2}$$

It is assumed that an aircraft will nominally remain in the same stream, and so the default values of the divergence parameters are

$$\beta_{ijmn}(k) = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases} \tag{3}$$

Figure 3 illustrates the model of a stream in an SEL. The dynamics of the air traffic flow in an SEL can be described using the inertia parameters and the divergence parameters, through the principle of conservation of aircraft. For instance, the difference equation describing the air traffic flow in the easterly stream in the surface element $(i,\ j)$ can be derived as[2,3]

$$x_{(i,j,3)}(k+1) = a_{ij33}(k) \sum_{m=1}^{8} \beta_{ij3m}(k) x_{(i,j,m)}(k) + \tau u_{(i,j,3)}(k)$$
$$+ \tau y_{(i,j-1,3)}(k) + \tau q^{\text{depart}}_{(i,j,3)}(k) + \tau q^{\text{exo}}_{(i,j,3)}(k) \tag{4}$$

In this equation, $x(k)$ denotes the number of aircraft in the stream at the sample instant k, $u(k)$ are the aircraft flow rates held back in the stream through flow control actions, $y()$ is the air traffic flow rate from the neighboring SEL, q^{depart} is the air traffic flow rate entering the SEL and joining the stream from airports under the SEL, and q^{exo} is the air traffic flow rate entering the stream from outside the modeled airspace, excluding departure traffic. The flow rates q^{exo} are inputs that are not controlled, whereas the departures q^{depart} may be controlled. The control variables in this equation are the air traffic flow rates $u(k)$ for metering actions and can also include the departure traffic flow rates q^{depart} from the airports under the SEL. Note that flow rates q are always positive by physical constraint and that the controls u are always positive by convention (flow subtracted from the output of the stream and added back to the input).

The en route output equations for an SEL can be written as

$$y_{(i,j,m)}(k) = \left[\frac{1 - a_{ijmm}(k)}{\tau}\right] \sum_{n=1}^{8} \beta_{ijmn}(k) x_{(i,j,n)}(k) - u_{(i,j,m)}(k), \qquad m = 1, 2, \ldots, 8 \tag{5}$$

Moreover, the landing air traffic flow rates into the airports under the SEL are given by

$$y_{(i,j,m)}(k) = \frac{1}{\tau} \sum_{n=1}^{8} \beta_{ijmn}(k) x_{(i,j,n)}(k), \qquad m = 9 \tag{6}$$

Several SELs are required to model realistic airspaces. In the present work, the numbering convention of the SELs (i, j) is that the index j is increasing from left to right, in the easterly direction, and i is increasing from bottom to top, in the northerly direction. In this work, the surface elements are used to model class A airspace (from 18,000 to 60,000 ft) (Ref. 12). Air traffic flow models of several SELs can be combined to form the overall Eulerian model of the airspace and can expressed in a compact form as

$$x(k+1) = A(k)x(k) + Bu(k) + B_d q^{\text{depart}}(k) + B_e q^{\text{exo}}(k) \tag{7}$$

The departure traffic may be subdivided according to those airports where they will be controlled by a ground delay program and where they will not. It is assumed that external traffic q^{exo} cannot be controlled directly. If the controlled inputs are combined into a vector $\boldsymbol{v}(k)$, and all other inputs are collected together into a disturbance vector $\boldsymbol{w}(k)$, the dynamic equation for the airspace is of the form

$$\boldsymbol{x}(k+1) = A(k)\boldsymbol{x}(k) + B_1\boldsymbol{v}(k) + B_2\boldsymbol{w}(k) \tag{8}$$

The state vector $\boldsymbol{x}(k)$ can be initialized using traffic data and then propagated forward in time. These equations can be used to facilitate analysis and synthesis of flow control strategies. Typically, not all states are of interest for analysis or for flow control. An output equation can be formulated to provide the variables of interest as

$$y(k) = C(k)\boldsymbol{x}(k) + D_1\boldsymbol{v}(k) \tag{9}$$

The Eulerian air traffic flow model consists of the time-varying difference equation for the state vector and the time-varying algebraic equation for the output vector. These equations can be formulated for SELs in any desired region of the NAS and combined to form a basis for analysis and flow-control system design.

Whereas the Eulerian modeling process is intuitively simple to carry out, it is impractical to derive these models manually for airspaces containing more than a few SELs. During the present research, a computer-aided modeling technique has been developed to derive Eulerian models of arbitrary dimension automatically using the FACET software as the traffic propagation engine.

When started with a specification of the airspace boundaries, SEL size, and sample time interval, the first step in the modeling process is that of determining the location of every aircraft in the airspace with respect to the SEL grid. Within each SEL, the heading angle of aircraft is then used to sort them into one of the eight streams. As an additional criterion, this determination may also be based on the SELs they are likely to occupy at the end of the sample time interval. This provides the initial condition for the Eulerian model.

Next, the aircraft trajectories are propagated using the FACET software for one sample interval. The new locations of the aircraft in the SELs, together with the aircraft location data at the beginning of the sample interval, are then used to compute the inertia and divergence parameters for each SEL during the sample time duration. This process is repeated for the next sample time and so on, for the total time duration of interest.

Note that FACET uses the actual flight plans of the aircraft, along with changes made by air traffic control (ATC), to propagate their positions. The determination of the values of the coefficients and the state variables is simply a bookkeeping procedure; thus, without any control action, the model is exact at each sample time. The model will change in subsequent intervals when the controls are applied, and so the model needs to be updated in that case. The reason for using FACET to determine the traffic flow parameters, instead of statistics from historical data, is that the model will be more accurate for control purposes. The control strategy, to be discussed in the following sections, is meant to be used in real time with the most current information, and FACET, given current flight plans, can provide a sufficiently accurate prediction of the trajectories of the aircraft, which in turn produces a reliable Eulerian model. As long as FACET has the most current information, there are essentially no uncertainties in the model, within the given resolution, and all disturbances are measurable. Uncertainties arise if the data used to initialize FACET are not accurate, or if an aircraft deviates from its declared flight plan.

A flowchart of the automatic modeling methodology is shown in Fig. 4. The modeling algorithm has been implemented in the form of a software package called MAESTRO. When started with the specifications of a few parameters, this software package enables the user to construct models of arbitrary size. The software also incorporates linear algebraic algorithms[13] to help carry out controllability, observability, reachability, order reduction, decentralization, and covariance analysis. All of the results given in this paper were generated using this software package.

III. Model Predictive Air Traffic Flow Control

One of the objectives of the present research is to demonstrate the application of Eulerian models for the synthesis of closed-loop air traffic flow control algorithms. These algorithms can initially be used as decision support tools, and, as other airspace automation initiatives[4,5] mature in the future, they could be used in a more automated mode. A block diagram illustrating the components of the air traffic flow control system is shown in Fig. 5.

Closed-loop air traffic flow control logic helps to achieve the desired traffic flow rates at arrival airports and to keep local traffic densities within limits in the national airspace by regulating the departures at airports and by modulating the flow through metering

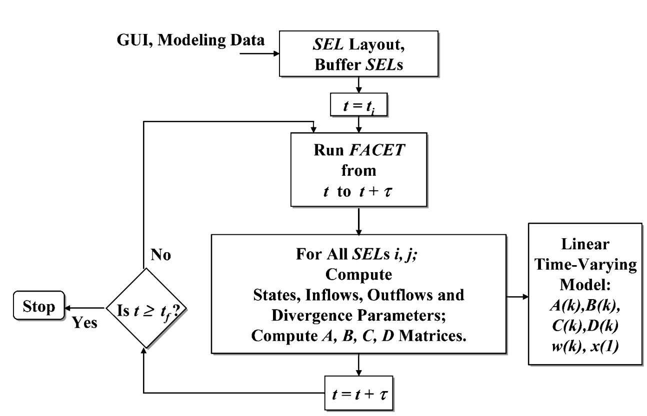

Fig. 4 Flowchart of the automatic Eulerian air traffic flow modeling methodology.

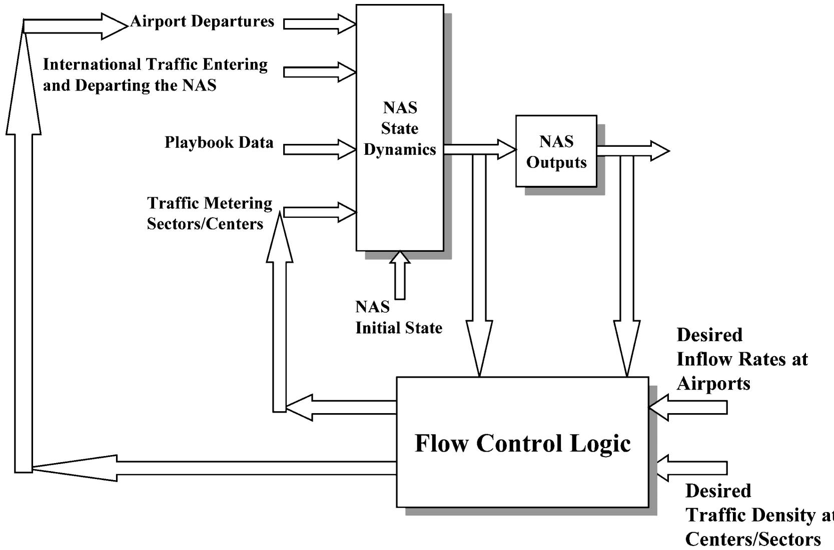

Fig. 5 Air traffic flow control system.

regions in the airspace. If the traffic density is low except at isolated time intervals and the flow control problems are localized, effective flow control can be achieved using simple strategies. However, as the traffic density increases, purely heuristic approaches may result in undesirable flow fluctuations in the airspace, and tools for flow control become important. Note that the traditional approach to air traffic flow control in the NAS is primarily through heuristic means.

Although sophisticated decision support tools[14] have been developed to manage aircraft trajectories, computational tools to manage traffic flows have not yet reached comparable levels of maturity. This section will demonstrate how the Eulerian models can be used to design en route flow control strategies. Because of the high order of the system's dynamics, its time-varying nature, and the constraints, the air traffic flow control is a challenging problem. Although it is feasible to consider the application of techniques such as linear-quadratic optimal control theory, the high order and the presence of multiple inequality constraints will make their use awkward in this problem. Ideally, an ATC strategy should be to able handle the entire NAS. For the present research, the MPC technique[9–11] was chosen for the flow control algorithm synthesis. MPC strategies have been in use for a number of years in the control of chemical processing plants, where there are often a large number of state variables, inputs, and outputs, along with constraints. Interestingly, the update rate for some of these control systems is on the same order as that anticipated in the air traffic flow control problems.

The basic idea in the MPC technique is to use a model of the system to predict the outputs up to N steps ahead (prediction horizon) using a nominal control policy. Nominal control policies are often adopted as either zero or constant values of control, subject to the control constraints. Next, an optimization problem is solved to determine the values of control that will minimize the error between the actual and the desired values of the outputs over the prediction horizon.

The Eulerian air traffic flow model over multiple time steps can be used to assemble an output predictor readily as

$$\tilde{y} = M_x x(k) + M_u \tilde{u} + M_d \tilde{q}^d \tag{10}$$

where

$$\tilde{y} = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+N) \end{bmatrix}, \qquad \tilde{u} = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N) \end{bmatrix}$$

$$\tilde{q}^d = \begin{bmatrix} q^{\text{depart}}(k) + {}^{\text{exo}}(k) \\ q^{\text{depart}}(k+1) + {}^{\text{exo}}(k+1) \\ \vdots \\ q^{\text{depart}}(k+N) + q^{\text{exo}}(k+N) \end{bmatrix} \tag{11}$$

$$M_x = \begin{bmatrix} C_0 \\ C_1 A_0 \\ C_2 A_1 A_0 \\ C_3 A_2 A_1 A_0 \\ \vdots \\ C_N A_{N-1} \quad A_0 \end{bmatrix}$$

$$M_u = \begin{bmatrix} D_0 & 0 & 0 & 0 & \cdots & \\ C_1 B_0 & D_1 & 0 & 0 & \cdots & \\ C_2 A_1 B_0 & C_2 B_1 & D_2 & 0 & \cdots & \\ C_3 A_2 A_1 B_0 & C_3 A_2 B_1 & C_3 B_2 & D_3 & & \\ \vdots & & & & \ddots & \\ C_N A_{N-1} \cdots B_0 & \cdots & & & & D_N \end{bmatrix} \tag{12}$$

$$M_d = \begin{bmatrix} 0 & 0 & 0 & 0 & \\ \tau C_1 & 0 & 0 & 0 & \\ \tau C_2 A_1 & \tau C_2 & 0 & 0 & \\ \tau C_3 A_2 A_1 & \tau C_3 A_2 & \tau C_3 & 0 & \\ \vdots & & & & \ddots \\ \tau C_N A_{N-1} \quad A_1 & \ldots & & & 0 \end{bmatrix} \tag{13}$$

where A_n, B_n, C_n, and D_n are from Eqs. (8) and (9) at time step $k+n$. A set of performance variables y_{perf} is defined next. These performance variables represent the traffic flows that the MPC algorithm expects to control, and they can be individual air traffic flows in specific SELs or linear combinations of traffic flows into airports or regions of interest in the en route airspace.

The essence of MPC is that, at each time step over the prediction interval, it is desired to minimize the difference between the actual values of the performance variables and the desired or commanded values y_d using the control variables. The variables comprising y_{perf} and its desired values y_d are selected based on the specific air traffic flow control objectives.

For the present work, the 1-norm is a suitable choice for the minimization problem in terms of the performance variables and their desired values. As a consequence, the optimization problem can then be cast as a linear programming problem. The objective of the air traffic flow control problem is to minimize

$$\|\tilde{y}_d - \tilde{y}_{\text{perf}}\|_1 = \sum_{i=1}^{(N+1)p} \left|\tilde{y}_{d_i} - \tilde{y}_{\text{perf}_i}\right| \tag{14}$$

with p being the number of performance variables included in the flow control problem. Note that the performance variables can be cast as $y_{\text{perf}} = Y_p y$ for some matrix Y_p, and $\tilde{y}_{\text{perf}} = \tilde{Y}_p \tilde{y}$ where $\tilde{Y}_p$ is a block-diagonal matrix with Y_p comprising the $N+1$ blocks. With this definition, the expression for the performance variables can be written as

$$\tilde{y}_{\text{perf}} = \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_u \tilde{u} + \tilde{Y}_p M_d \tilde{q}^d \tag{15}$$

In this case, the linear programming problem can then be expressed as

$$\min_{\gamma,\tilde{u}} [\mathbf{1} \quad \mathbf{0}] \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix}$$

subject to

$$\begin{bmatrix} I & \tilde{Y}_p M_u \\ I & -\tilde{Y}_p M_u \end{bmatrix} \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix} + \begin{bmatrix} \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_p M_x x(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \end{bmatrix} \geq \mathbf{0} \tag{16}$$

The vector γ consists of bounding variables, one variable for each term in the 1-norm. Note that the symbol $\mathbf{1}$ in the cost function represents a row vector of ones and $\boldsymbol{I}$ in the inequality is an identity matrix.

Additional constraints in the problem are that the controls must be greater than or equal to zero to be physically meaningful. Because the controls are part of the solution vector, the lower bounds can be handled directly. Another constraint is that the outflows in the streams where metering is taking place must be greater than or equal to zero, which in effect defines the upper bounds on the controls. Because these upper bounds are dependent on the state of the system, they cannot be specified directly and must be included as constraint equations. Let these constrained outputs be defined as $y_c = Y_c y$ with Y_c being a matrix of zeros and ones, and let $\tilde{y}_c = \tilde{Y}_c \tilde{y}$, where $\tilde{Y}_c$ is a block-diagonal matrix with Y_c comprising the $N+1$ blocks. The additional constraint equation is

$$[0 \quad \tilde{Y}_c M_u] \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix} + \left[\tilde{Y}_c M_x \boldsymbol{x}(k) + \tilde{Y}_c M_d \tilde{q}^d\right] \geq \mathbf{0} \tag{17}$$

This constraint equation augments the preceding equations. The linear programming problem that the software used here solves is actually

$$\min_z c^T \boldsymbol{z}, \qquad \text{subject to} \quad F\boldsymbol{z} = g, \quad \boldsymbol{z}_{\text{lower}} \leq \boldsymbol{z} \leq \boldsymbol{z}_{\text{upper}} \tag{18}$$

The vector $\boldsymbol{z}$ contains the unknowns and $(\,)^T$ represents the transpose of a vector or matrix. Slack variables are introduced as in standard linear programming problems to transform the inequality constraints into equality constraints. The linear programming problem for the MPC problem is then formed as follows. Let the vector of unknown quantities be

$$\boldsymbol{z}^T = [\gamma^T \quad \tilde{u}^T \quad \boldsymbol{r}^T \quad \boldsymbol{s}^T \quad \boldsymbol{t}^T] \tag{19}$$

where $\boldsymbol{r}$, $\boldsymbol{s}$, and $\boldsymbol{t}$ are column vectors of slack variables of dimensions $(N+1)p$, $(N+1)p$, and $(N+1)m$, respectively, where m is the number of control inputs,

$$c^T = [\mathbf{1} \quad \mathbf{0}] \tag{20}$$

where $\mathbf{1}$ is length $(N+1)p$ and $\mathbf{0}$ is length $(N+1)(2m+2p)$ and

$$F = \begin{bmatrix} I & \tilde{Y}_p M_u & -I & 0 & 0 \\ I & -\tilde{Y}_p M_u & 0 & -I & 0 \\ 0 & \tilde{Y}_c M_u & 0 & 0 & -I \end{bmatrix}$$

$$g = \begin{bmatrix} -\tilde{Y}_p M_x \boldsymbol{x}(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \\ \tilde{Y}_p M_x \boldsymbol{x}(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_c M_x \boldsymbol{x}(k) - \tilde{Y}_c M_d \tilde{q}^d \end{bmatrix} \tag{21}$$

For the controls, the lower bounds are zero, and although the upper bounds are determined by constraints, a value of 50 aircraft/step was specified as a practical measure. The lower bounds on the slack variables are set to zero, and the upper bounds are set to a large number, 10^{32}, in the present work. Likewise, the lower bounds on the bounding variables are set to zero, and the upper bounds are set to a large number, 10^{32}.

In the case where departure controls are included, the appropriate columns of $\tilde{y}_p M_d$ appear in the F matrix on the left-hand side of the constraint equation, and an additional constraint is needed. Whereas the requirement that the output must be nonnegative could be used, in this case the maximum value of the control is determined by the number of departures in that stream where control is being applied. This gives a simpler expression to implement. Let $\hat{\tilde{q}}^d$ be the subset of departures where control is to be applied, and let $\tilde{u}^d$ be the subset of the controls applied to the departures. Then the constraint is

$$\hat{\tilde{q}}^d - \tilde{u}^d \geq 0 \tag{22}$$

Because this is an inequality, another set of slack variables must be added so that the vector of unknowns becomes

$$\boldsymbol{z}^T = [\gamma^T \quad \tilde{u}^T \quad \boldsymbol{r}^T \quad \boldsymbol{s}^T \quad \boldsymbol{t}^T \quad \boldsymbol{v}^T] \tag{23}$$

For simplicity of notation, it will be assumed that the control vector is partitioned so that the metering controls for the prediction horizon $[k, k+N]$ are first, and the departure controls over the same interval are second, that is,

$$\tilde{u} = \begin{bmatrix} \tilde{u}^m \\ \tilde{u}^d \end{bmatrix} \tag{24}$$

Let $\hat{M}_d$ be the matrix composed of the columns of M_d corresponding to the departures where control is applied. Then the left- and right-hand sides of the constraint equations are of the following form:

$$F = \begin{bmatrix} I & \tilde{Y}_p M_u & \tilde{Y}_p \hat{M}_d & -I & 0 & 0 & 0 \\ I & -\tilde{Y}_p M_u & -\tilde{Y}_p \hat{M}_d & 0 & -I & 0 & 0 \\ 0 & \tilde{Y}_c M_u & -\tilde{Y}_c \hat{M}_d & 0 & 0 & -I & 0 \\ 0 & 0 & -I & 0 & 0 & 0 & -I \end{bmatrix}$$

$$g = \begin{bmatrix} -\tilde{Y}_p M_x \boldsymbol{x}(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \\ \tilde{Y}_p M_x \boldsymbol{x}(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_c M_x \boldsymbol{x}(k) - \tilde{Y}_c M_d \tilde{q}^d \\ -\hat{\tilde{q}}^d \end{bmatrix} \tag{25}$$

The linear programming problems for various air traffic flow control situations formulated in this section are solved using a software package called PCx from Argonne National Laboratory.[15] This software has been integrated into the Eulerian modeling software MAESTRO mentioned in the preceding section.

A flowchart of the model predictive air traffic flow control algorithm is shown in Fig. 6. Because the coefficients of the Eulerian model used for MPC are derived from the traffic data, the application of controls will change the traffic flow. This will in turn cause changes in the model. To synthesize correct control decisions for the next sample, the model coefficients must be recomputed using the

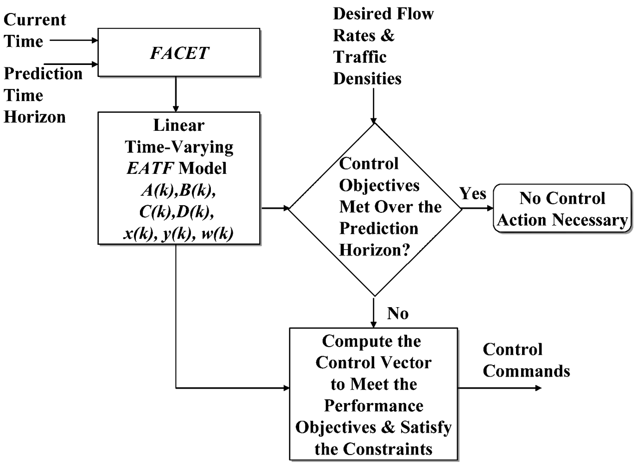

Fig. 6 Model predictive air traffic flow control using Eulerian model.

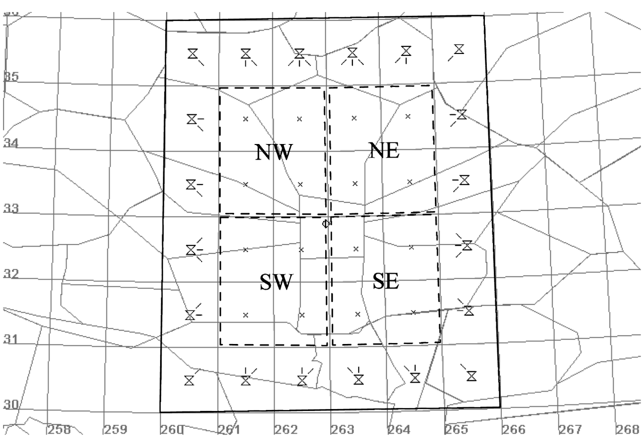

Fig. 7 Metering controls in the vicinity of DFW Airport.

air traffic resulting from the application of controls in the preceding sample. Thus, every control action must be followed by the recomputation of the model coefficients, that is, FACET is rerun with the controls in place. The new model must then be used in the MPC flow control methodology.

The following subsections will illustrate two different air traffic flow control examples using the MPC methodology.

A. Example 1: Dallas–Fort Worth Area Metering

The objective of this example is to regulate the air traffic flow descending into the area surrounding the Dallas–Ft. Worth (DFW) Airport for the time period between 1200 and 1400 hrs on a typical day. Figure 7 illustrates the region under consideration. The SELs included in the control problem are outlined. The airport is represented near the middle of Fig. 7. The surface elements are 1° by 1°, or approximately 60 n miles north–south by 50 n miles east–west. The sample time interval is 6 min. The metering locations are indicated by hourglass symbols and the directions are indicated by short line segments in Fig. 7. There are a total of 40 controls, which were selected in part by observing the traffic flow over the specified period. The Eulerian traffic flow model has 512 state variables. (Although 36 SELs are shown, a boundary layer of SELs around the perimeter of the area is added for computational purposes, giving 64 SELs $\times$ 8 streams $=$ 512 state variables.) The arrivals into the DFW area are assumed to be those aircraft descending from class A airspace in the 16 innermost SELs. For simplicity, the descending traffic streams are summed by quadrants (NW, NE, SE, and SW) as outlined with dashed lines in Fig. 7, resulting in four outputs to be controlled.

The time histories of the outputs without flow control inputs are shown in Fig. 8. Note that the flow rates do not follow any particular pattern. The MPC strategy is implemented next, with the requirement that the desired flow rate for each quadrant be less than or

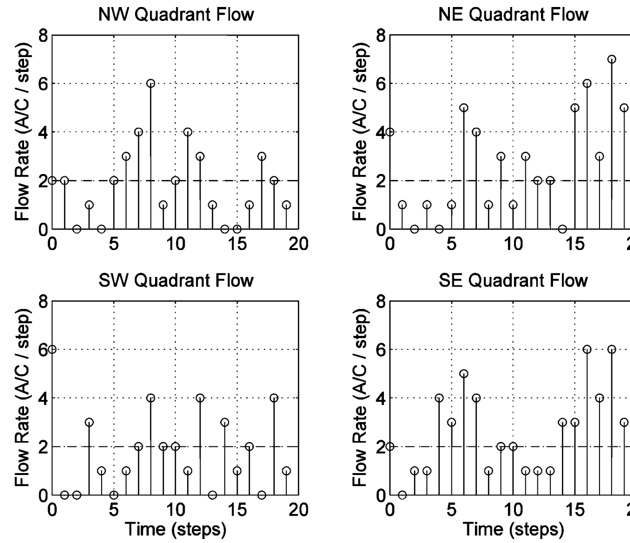

Fig. 8 Air traffic flows into DFW Airport area without metering.

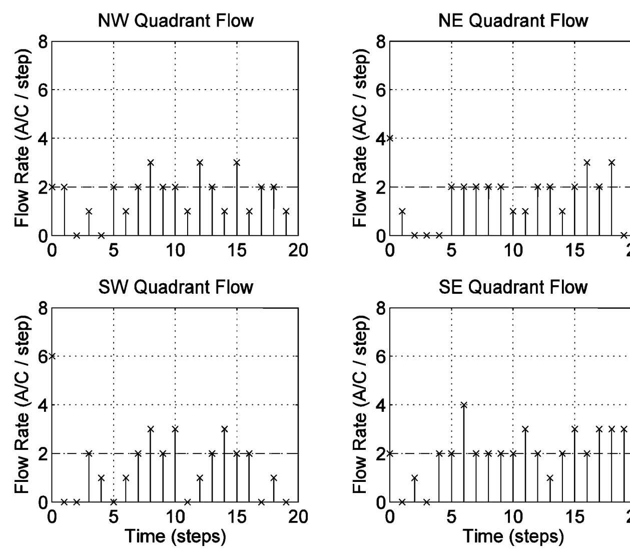

Fig. 9 Air traffic flows into DFW Airport area with metering.

equal to two aircraft/step. The prediction horizon is four time steps. The time histories of the aircraft flows for the four quadrants under closed-loop control are shown in Fig. 9. Note that the flow rates are much more regular but the control objectives are not strictly satisfied at all time steps. Note that it is not possible to achieve exact control because the metering controls are constrained and that the flow is not metered in all possible directions.

In the foregoing discussions, the aircraft flows in four separate zones were regulated. An alternative control objective is the regulation of the total traffic flow into the DFW area. In view of this, the MPC problem is reformulated with one output defined as the sum of all 16 landing outputs and requiring the desired flow rate to be less than or equal to eight aircraft/sample. The model predictive flow control then produces the time history shown in Fig. 10. Figure 10 also indicates the aircraft flow without control.

As in the preceding case, the present control objective is strictly met only at certain samples. However, the flow rate much more regular under closed-loop control. Achieving a better flow control may require the introduction of additional metering SELs in the modeled region and/or more metering directions in existing SELs.

B. Example 2: Air Traffic Density Control in a Region

A flow control problem that sometimes arises in the NAS is that of maintaining the density of air traffic in certain regions below a certain specified level to limit the workload on the human air traffic

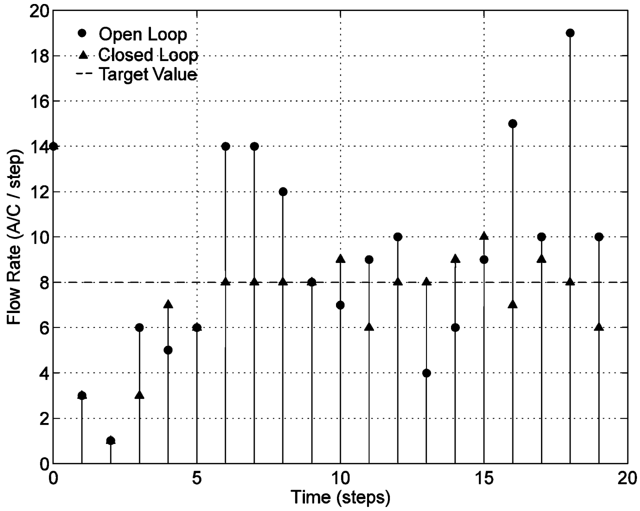

Fig. 10 Air traffic flow rate into DFW Airport area under metering.

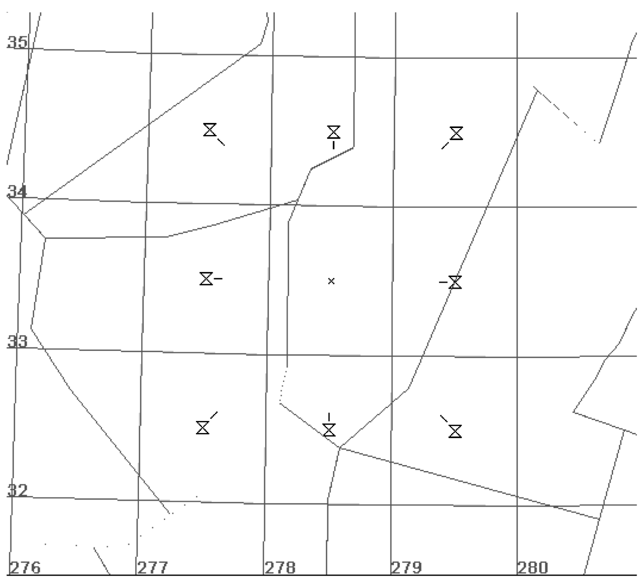

Fig. 11 SEL considered in traffic density control example.

controller. This example illustrates how the MPC methodology can be used for traffic density control. For illustrative purposes, in this example, the aircraft density in a single surface element will be regulated. Extending this approach to multiple surface elements is straightforward.

The traffic density is defined in this paper as the total number of aircraft within an SEL at a sample instant. Because the airspace volume represented by the SEL is known, the actual traffic density/unit volume and the total aircraft count are proportional. The density is computed by summing the aircraft in each of the eight streams in the SEL of interest. For the present example, the SEL of interest is between 33° and 34° north latitude and 82° and 83° west longitude, an area east of Atlanta's Hartsfield Airport. This region was selected arbitrarily among those that appeared to have a steady flow of traffic. Metering points were chosen at the streams in neighboring SELs that feed into the controlled SEL. The SEL of interest is shown in Fig. 11. Figure 11 has 200 states (including the boundary layer) and 8 controls.

Note that metering controls have been placed in neighboring SELs in directions pointing toward the SEL. The desired density is four aircraft or less within the SEL, and the prediction horizon is two time steps (6 min/step). A comparison of the controlled and uncontrolled densities is shown in Fig. 12. The MPC-based controller is able to maintain the desired density to within two aircraft or less and, for the most part, one or less.

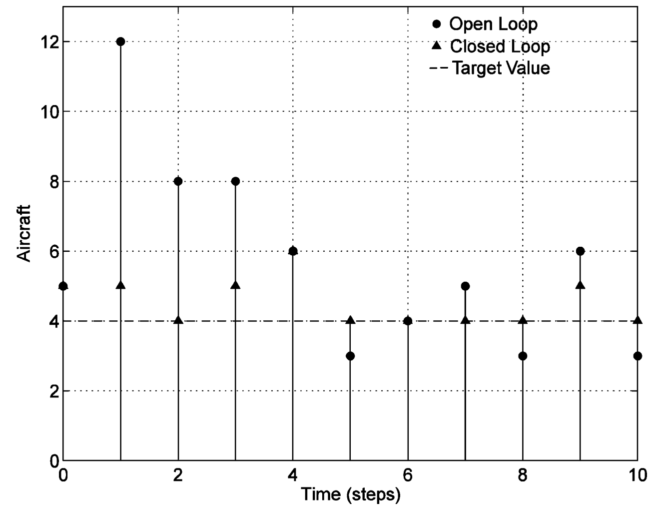

Fig. 12 Density control results.

The two examples given in this section illustrate the MPC-based air traffic flow control logic synthesis. Although the performance of the MPC controllers was satisfactory, significant improvements are possible through additional analysis of the Eulerian models.

Validation of the model and the control algorithm remains an issue. To validate the Eulerian model, actual data could be used and compared with predictions from the linear models, similar to the methodology in Ref. 16. Validation of the control actions presents additional challenges. One way might be to obtain the actual data flight, including the original flight plans and the metering subsequently applied by ATC, and then to simulate the same scenario and compare the controls generated by the algorithm with the actual controls. Note that the proposed approach deals only with the gross behavior of the aircraft as they move from one SEL to another and does not determine specific speed and/or heading commands that would be used to achieve metering.

One possible limitation of the proposed approach is that it models the airspace using a rectangular grid, whereas en route air traffic control is currently implemented by sectors, that are far from rectangular. Therefore, the control actions generated by the present method may have some ambiguity in where they should be applied in the existing sector layout. The present modeling methodology can be refined to handle this situation by using finer grid size and defining additional output equations.

IV. Conclusions

This paper presented the development of a computer-aided Eulerian air traffic flow modeling methodology and its application to deriving quantitative flow control strategies. The flow control algorithms can initially be used as decision support tools, and, as other airspace automation initiatives mature in the future, they could possibly be used in a more fully automatic mode.

The Eulerian modeling methodology divides the airspace into interconnected SELs, and the dynamics of the traffic flow through and between these SELs is then derived by invoking the principle of conservation. Although the Eulerian approach preserves no information on the motion of individual aircraft, it provides a convenient formalism for aggregating air traffic flow information. An automatic procedure for deriving the Eulerian models from individual aircraft trajectories was developed during the present research. In this approach, the user provides inputs such as the region of the national airspace to be modeled, spatial discretization, sample time, metering locations, airports subject to departure control, the output locations, and the time interval of interest. The automatic modeling procedure then uses the FACET software to assemble the Eulerian model. A software package MAESTRO has been developed and integrated into FACET to assist the user in automatically constructing Eulerian air traffic flow models.

By aggregation of the traffic information in the form of discrete-time, linear time-varying models, the Eulerian model enables several types of useful analyses of the traffic flow in the airspace, as well as feedback control methods. The MPC technique was employed in conjunction with the Eulerian model to synthesize air traffic flow control algorithms. The MPC technique uses the Eulerian model to make predictions over a specified time horizon about the future values of performance variables under a nominal control policy. An optimization technique is then used to refine the nominal control policy to achieve the desired values of states and outputs.

The present research considered the 1-norm of the error between desired and predicted performance variables as the performance index. Inequality constraints were specified on the control variables and output variables. Because Eulerian models are linear, the resulting optimization problem is in the form of a linear program. This linear programming problem was then solved using a well-known software package. Flow control synthesis for two different control objectives was then demonstrated.

This paper demonstrated that it is feasible to use Eulerian air traffic flow models for analysis and flow control system synthesis. The methodology given here can be readily tailored to practical traffic flow control problems to improve the efficiency of en route air traffic flow control in the next-generation ATC system. Investigation of alternate control techniques and model accuracy improvements will be of future interest.

Acknowledgments

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